III. PROJECTILE MOTION

Three cases are considered: P1, the projectile is accelerated in the $+x$ direction for explosive configuration E1 with $b > a$; P2, the projectile is accelerated in the $-x$ direction for explosive configuration E1 with $b < 0$; P3, the projectile is accelerated in the $+x$ direction for explosive configuration E2 with $b > a$. For $\gamma = 3$ Eq. (1) can be written

$$y'' = \pm 8Q(c/D)(u/D - y')^2 \tag{7}$$

where $y = s/a$, $z = Dt/a$, $Q = 2K\rho_0 Aa/9m$, and $y' \equiv dy/dz$. Initial conditions are $y = y_0$, $y' = 0$ at $z = z_0$. The three cases are distinguished by the expressions for u and c, given in the previous section.

A. Case P1, $b > a$

The projectile path is the dotted curve bH in Figure 1. Substituting Eqs. (3) into (7) with $x = s$ yields the equation to be solved for y:

$$y'' = Q[y/z - (y - 1)/(z - 1)] \\ [y/z + (y - 1)/(z - 1) - 2y']^2 \tag{8}$$

Motion of the projectile lies entirely within Region II since $u > 0$ for all $t > a/D$. Moreover the acceleration is never negative: $u > ds/dt$ initially and the difference diminishes as the projectile accelerates and the velocity of its local environment changes. When $u = ds/dt$, the projectile is in a region of constant particle velocity with no forces acting on it, so it will continue in that state indefinitely.

Eq. (8) has been integrated numerically for various values of $b/a > 0$ and for various Q. The results are shown in Figures 5 and 6 and in Table I. The terminal velocity is taken to be the last value obtained in the numerical integration, usually at $z > 10.0$. The effect of varying y_0 is illustrated in Table I. There is an appreciable increase of final velocity with y_0, the terminal velocity increasing as y_0 increases and the change being greater for small Q than for large. As y_0 approaches unity Eq. (7) becomes meaningless because it ignores the finite size of the object accelerated; it also becomes singular.

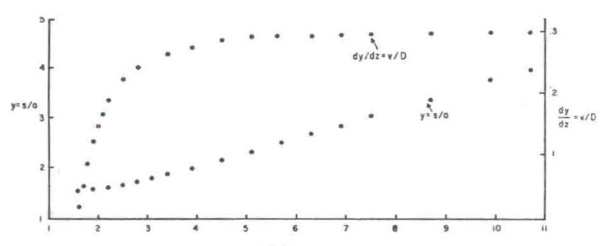

Figure 5

Trajectory and velocity of projectile for case P1; $b = 1.5a$, $Q = 1.0$.

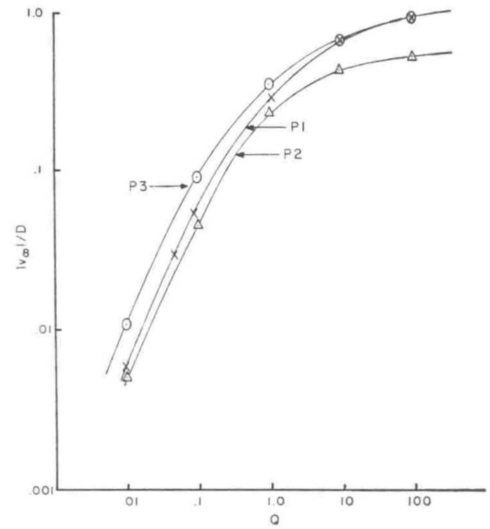

Figure 6

Terminal velocities of explosively accelerated projectiles. Initial positions: P1 and P3, $b/a = 1.1$; P2, $b/a = -.01$.

TABLE I

TERMINAL VELOCITIES OF EXPLOSIVELY ACCELERATED PROJECTILES

$y_0 = x_0/a = -.01$ FOR $P2$. $v_\infty = D(dy/dz)_{z=\infty}$

	P1, P3	$\|v_\infty\|/D$			e		
Q	x_0/a	P1	P3	P2	P1	P2	P3
.01	1.1	.00587	.0109	.00503	1.09	.0015	1.06
	1.5	.00633					
.05	1.5	.0292					
.10	1.1	.0530	.0884	.0448	.989	.194	.766
	1.5	.0572					
1.0	1.01	.261					
	1.1	.284	.338	.220	.632	.408	.124
	1.5	.299	.351				
10.0	1.1	.632	.632	.408	.268	.808	.267
	1.5	.649					
100.0	1.1	.848	.848	.480	.103	1.50	.104
	1.5	.858					

Trajectories for all values of Q listed in Table I are similar to that shown in Figure 5, except that the final velocity is approached more rapidly for larger Q. In each case the trajectory is asymptotic to a straight line, $y = e + y_\infty' z$, where y_∞' is the asymptotic value of dy/dz. Terminal velocities for $y_0 = 0.0$ are plotted as curve P1 in Figure 6; values of e are given in Table I.

B. Case P2

The projectile path is the dotted curve JK of Figure 1. The equation of motion in Region I is obtained by substituting Eqs. (2) into Eq. (7). Defining y and z as before yields:

$$y'' = -Q(y/z + 1/2)(y/z - 1/2 - 2y')^2 \qquad (9)$$

Eq. (8) with a change of sign still applies in Region II. The projectile starts out in Region I and remains there so long as $(y - 1)/(z - 1) < -1/2$. When $(y - 1)/(z - 1) > -1/2$, Eq. (9) applies. When the projectile crosses the boundary between I and II, y and y' are continuous. Initial conditions are $y = b/a$, $y' = 0$ at $z = 2b/a$, $b < 0$. Terminal velocities are shown in Figure 6 and Table I.

C. Case P3

The projectile path is the dotted curve bJK of Figure 3. Eq. (8) is the equation of motion in Region II. In Region IV:

$$y'' = Q[1/2 - (y - 1)/(z - 1)] [1/2 + (y - 1)/(z - 1) - 2y']^2. \qquad (10)$$

In Region V:

$$y'' = 8Q[y/(z - 1) - y']^2/(z - 1) \qquad (11)$$

Terminal velocities obtained from numerical integration are shown in Figure 6 and Table I. The transition from Region II to Region IV occurs when $y/z = 1/2$; that from IV to V when $(y + 1)/(z - 1) = 1/2$. When Q is large the projectile may not pass into Region IV at all, or may pass into it at such a late time that the event is no longer of interest or significance so far as its final velocity is concerned. It is this division of projectiles according to Q which produces the minimum in e for P3, Table I. For $Q \leqq 1.0$, the projectile passes into IV and V; for $Q \geqq 10$ it remains in II. This possibility can be inferred from Figure 4. Values of y and z can be transformed to (u, c) pairs and plotted to yield the curve cvq for $Q = 10$. Physically, this occurs because the particle first enters Region II where $u = D$ and $\rho = 0$, experiencing little acceleration. As time passes, ρ increases and u decreases but is still much larger than v. Then as time increases still more, ρ begins to decrease because the detonation gases have blown past the particle or are traveling backward. Moreover $u \to v$ so the acceleration is doubly-diminished. At the point q in Figure 4 the projectile has reached its terminal velocity, equal to the value of u at q. The form of this curve suggests that it would be useful for estimating y_∞'.

IV. DISCUSSION

General features of the results are shown in Figure 6. If one seeks maximum velocity, the projectile should be placed ahead of the explosive: cases P1 and P3. At lower velocities a rigid backing gives some additional impulse to the projectile, but, in each case calculated, is less effective than doubling the explosive thickness with no backing.

TABLE II

TERMINAL VELOCITY OF 200 MICRON SPHERE ACCELERATED BY EXPLOSIVE CYLINDER WITH $L/d = 3$; $d \equiv a$. $Q = a/30d$

a, cm	*Q*	*v/D*	*v m/sec*	*Explosive mass*
1	1.7	.35	3100	3.8 gm
10	17	.68	6000	8.5 lb
100	170	.89	7800	4.25 t

Detonation velocity = 8800 m/sec

The significance of the results can be better realized if we relate them to a particular experiment. Suppose a steel sphere of 200 microns diameter is to be accelarated by an explosive cylinder for which length/diameter equals three. Assume that the effective thickness of the equivalent slab is one diameter. Then the relation between explosive mass and terminal velocity is as shown in Table II. This shows clearly that terminal velocity increases so slowly with explosive mass that